UNIT-2

LINEAR PROGRAMMING PROBLEM- FORMULATION –GRAPHICAL METHOD- SIMPLEX METHOD

Introduction, Basic Assumptions, Structure of Linear Programming model, Formulation of LPP, Solution by Graphical method: Multiple Optimal Solutions, Unbounded Solution, Infeasible Solution; Simplex method, Applications, Advantages, Limitations.

Course Outcome: *Demonstrate the concept of Linear Programming Problem and its use in management decisions for optimization.*

Chapter Outlines:

- Introduction and Assumptions of Linear Programming Problem (LPP)
- General Structure of LPP
- Formulation of LPP
- Graphical Solution of LPP and special cases
- Simplex method of LPP

INTRODUCTION:

A large number of decision problems faced by a business manager involve allocation of resources to various activities, with the object of increasing profits or decreasing cost. Normally, the resources are scarce and performance of number of activities within the constraints of limited resources is the challenge. A manager is, therefore, required to decide as to how best to allocate resources among the various activities.

The mathematical programming involves optimisation of a certain function, called *objective function,* subject to the given limitations or *constraints*. A manager may be faced with the problem of deciding the appropriate product mix taking the objective function as the maximising of profits obtainable from the mix, keeping in view various constraints such as availability of raw materials, position of labour supply, market consumptions etc.,

LINEAR PROGRAMMING PROBLEM

Linear programming deals with the optimization of a function of variables known as objective functions. It is subject to a set of linear equalities and/or inequalities known as constraints. Linear programming is a mathematical technique which involves the allocation of limited resources in an optimal manner, on the basis of a given criterion of optimality.

A Linear Programming Problem (LPP) consists of three components, namely (i) Decision Variables (activities), (ii) The Objective (goal) and (iii) The constraints (restrictions).

ASSUMTIONS OF LPP: The following four basic assumptions are necessary for all linear programming problems:

(a) Certainty: Certainty in linear programming refers to the assumption that the parameters of the objective function coefficients and the coefficients of constraints are known with certainty. For example, profit per unit of product, resource availability per unit, etc. are known with certainty.

(b) Proportionality / Linearity: Linear programming assumes that any modification in the constraint inequalities will result in a proportional change in the objective function. This means that if it takes 10 hours to produce 1 unit of a product, then it would take 50 hours to produce 5 such products.

(c) Additivity: Additivity means that each function in a linear programming model is the sum of the individual contributions of the respective activities. For example, the total profit is determined by the sum of profit contributed by each activity separately.

(d) Divisibility: Linear programming makes the divisibility assumption that the solution has to be in whole numbers i.e. integers. This assumption means that decision variable may take any value, including non-integer values, as long as functional and non-negativity constraints are satisfied.

General Structure of LPP

The general formulation of the LPP can be stated as follows:

In order to find the values of n decision variables $x_1, x_2, \ldots, x_n$ to maximize or minimize the objective function

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \qquad \text{----------------------(1)}$$

And also satisfy m-constraints

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \; (\leq = \geq) \; b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \; (\leq = \geq) \; b_2 \\ \quad . \\ \quad . \\ a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \; (\leq = \geq) \; b_i \\ \quad . \\ \quad . \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \; (\leq = \geq) \; b_m \end{array}\right] \text{------------(2)}$$

Where constraints may be in the form of inequality $\leq$ or $\geq$ or even in the form an equation (=) and finally satisfy the non-negative restrictions

$$X_1 \geq 0, X_2 \geq 0 \ldots X_n \geq 0 \qquad \text{-------------------(3)}$$

FORMULATION OF LP PROBLEMS

The procedure for mathematical formulation of a LPP consists of the following steps:

Step 1: To write down the decision variables of the problem.
Step 2: To formulate the objective function to be optimized as a linear function of the decision variables.
Step 3: To formulate the other conditions of the problem such as resource limitation, market constraints, interrelations between variables.
Step 4: To add the non-negativity constraint from the considerations so that the negative values of the decision variables do not have any valid physical interpretation.

The objective function, the set of constraint and the non-negative constraint together form a Linear programming problem.

DEFINITIONS

***Solution of LPP*:** An n-tuple ($x_1, x_2, \ldots\ldots x_n$) of real numbers which satisfies the constraints of a general LPP is called a *solution* of LPP.

Feasible solution: Any solution to a general LPP which also satisfies non-negative restrictions of the problem, is called a *feasible solution* to the general LPP.

Optimum solution: Any feasible solution which optimizes (minimize or maximize) the objective function of general LPP is called an *optimum solution* the general LPP.

GRAPHICAL METHOD

The steps involved in graphical method are as follows.

Step 1: Consider each inequality constraint as equation.

Step 2: Plot each equation on the graph as each will geometrically represent a straight line.

Step 3: Mark the region, if the inequality constraint corresponding to that line is ≤ then the region below the line lying in the first quadrant is shaded. For the inequality constraint ≥ sign, the region above the line in the first quadrant is shaded. The points lying in common region will satisfy all the constraints simultaneously. The common region thus obtained is called the feasible region.

Step 4: Assign an arbitrary value say zero for the objective function.

Step 5: Draw the straight line to represent the objective function with the arbitrary value.

Step 6: Stretch the objective function line till the extreme points of the feasible region. In the maximization case this line will stop farthest from the origin and passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passing through at least one corner of the feasible region.

Step 7: Find the co-ordinates of the extreme points selected in step 6 and find the maximum or minimum value of Z.

IN SIMPLE WAY

The Graphical Method

Step 1: Formulate the LP (Linear programming) problem.

Step 2: Construct a graph and plot the constraint lines.

Step 3: Determine the valid side of each constraint line.

Step 4: Identify the feasible solution region.

Step 5: Plot the objective function on the graph.

Step 6: Find the optimum point.

SIMPLEX ALGORITHM (Procedure)

For the solution of any LPP by simplex algorithm the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as follows:

Step 1: Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of Maximization by Min Z= - Max (-Z)

Step 2: Check whether all b_i (i=1, 2….m) are positive. If any one of b_i is negative then multiply the inequation of the constraint by -1 so as to get all b_i to be positive.

Step 3: Express the problem in standard form by introducing slack/surplus variables, to convert the inequality constraints into equations.

Step 4: Obtain an initial basic feasible solution to the problem in the form $X_B = B^{-1} b$ and put it in the first column of the simplex table.

Step 5: Compute the net evaluations $C_j - Z_j$ by using the relation

$$C_j - Z_j = C_j - (C_B X_j)$$

Examine the sign of $C_j - Z_j$

(i) If all $C_j - Z_j \leq 0$, then the initial basic feasible solution X_B is an optimum basic feasible solution.

(ii) If at least one $C_j - Z_j > 0$, then proceed to next step as the solution is not optimal.

Step 6: (to find the entering variable i.e. 'key column')

If there are more than one positive of $C_j - Z_j$, choose the maximum of them. This gives the entering variable X_r. If there are more than one variable having the same maximum value of $C_j - Z_j$ then any one of the variables can be selected arbitrarily as the entering variable.

(i) If all $X_{ir} \leq 0$ (i=1,2……..m) then there is an unbounded solution to the given problem.

(ii) If at least one $X_{ir} > 0$ (i=1,2….m) then the corresponding vector X_r enters the basis.

Step 7: (To find the leaving variable or key row)

Compute the ratio (X_{Bi}/X_{kr}, $X_{ir}>0$)

If the minimum of these ratios be X_{Bi}/X_{kr}, then choose the variable X_k to leave the basis called 'key row' and the element at the intersection of key row and key column is called the key element.

Step 8: Form a new basis by dropping the leaving variable and introducing the entering variable along with the associated value under C_B column. Convert the leading element to unity and other elements in its column to zero.

Step 9: Go to step (5) and repeat the procedure until either an optimum solution is obtained or there is an indication of unbounded solution.

APPLICATIONS OF SIMPLEX METHOD

The simplex method is a mathematical technique that can be used in many applications, including:

- **Programming**

The simplex method can be used to solve linear programming (LP) problems that arise in programming.

- **Industrial planning**

The simplex method can be used to maximize profits or minimize resources needed in industrial planning.

- **Monitoring software**

The simplex method can be used to optimize resource allocation, task scheduling, and performance in monitoring software.

- **Software development**

The simplex method can be used to determine the optimal allocation of resources and maximize profit in software development.

- **Electrical engineering**

The simplex method can be used to optimize the distribution of electric stress in a gas insulated substation.

- **Accounting**

The simplex method can be used to solve accounting problems such as capital budgeting, establishing optimum transfer prices, and cost volume profit analysis.

- **Radiotherapy treatment**

The simplex method can be used in radiotherapy treatment to help find a treatment plan that reaches the dose lower limit of the tumour.

ADVANTAGES OF SIMPLEX METHOD

The simplex method has many advantages, including:

- **Efficiency**

It's efficient for solving linear programming problems, especially for large-scale problems with many variables and constraints.

- **Flexibility**

It can be used for both maximization and minimization problems, and it can adapt to multiple optimization goals.

- **Optimization**

It systematically examines the feasible solution space to find the optimal solution.

- **Decision-making**

It provides optimal solutions to support informed decision-making processes.

- **Wide applicability**

It can be used in many industries and areas, including business operations, economics, engineering, and research.

- **Easier to use than the graphical method**

It's easier to handle than the graphical method, which can only be used when there are two variables in the model.

- **Can be digitized and automated**

The simplex method can be digitized and automated.

- **Generates economic data**

It generates important economic data as a side effect.

LIMITATIONS AND POTENTIAL DRAWBACKS OF SIMPLEX METHOD

While the Simplex Method offers numerous benefits, it is essential to consider its limitations and potential drawbacks:

- **Nonlinear programming:** The Simplex Method is not suited to handle nonlinear programming problems. It is restricted to linear problems, prohibiting its application to problems with nonlinear constraints or objective functions.

- **Problem size:** Although the Simplex Method can handle large-scale problems, problems with a vast number of variables and constraints may become computationally intensive and time-consuming to solve. Advanced hardware or parallel computing may be required for very large-scale linear programming problems.

- **Convergence issues:** In rare occasions, the Simplex Method may lead to cycling, where the algorithm repeatedly pivots between a cycle of tableaus without making progress towards an optimal solution.

- **Sensitivity to input data:** A change in input data, such as the objective function coefficients, constraint coefficients, or the constraint constants, can affect the optimal solution. Sensitivity analysis is required to assess how changes in input data impact the optimal solution in real-world applications, where data is often updated or modified.

Examples of Linear Programming Model Formulation

Example 1

Universal Corporation manufactures two products- P_1 and P_2. The profit per unit of the two products is Rs. 50 and Rs. 60 respectively. Both the products require processing in three machines. The following table indicates the available machine hours per week and the time required on each machine for one unit of P_1 and P_2. Formulate this product mix problem in the linear programming form.

Machine	Product		Available Time (in machine hours per week)
	P_1	P_2	
1	2	1	300
2	3	4	509
3	4	7	812
Profit	Rs. 50	Rs. 60	

Solution.

Let x_1 and x_2 be the amounts manufactured of products P_1 and P_2 respectively. The objective here is to maximize the profit, which is given by the linear function

Maximize $z = 50x_1 + 60x_2$

Since one unit of product P_1 requires two hours of processing in machine 1, while the corresponding requirement of P_2 is one hour, the first constraint can be expressed as
$2x_1 + x_2 \leq 300$

Similarly, constraints corresponding to machine 2 and machine 3 are
$3x_1 + 4x_2 \leq 509$
$4x_1 + 7x_2 \leq 812$

In addition, there cannot be any negative production that may be stated algebraically as
$x_1 \geq 0, x_2 \geq 0$

The problem can now be stated in the standard linear programming form as

Maximize $z = 50x_1 + 60x_2$

subject to

$2x_1 + x_2 \leq 300$
$3x_1 + 4x_2 \leq 509$
$4x_1 + 7x_2 \leq 812$

$x_1 \geq 0, x_2 \geq 0$

Linear Programming Graphical Method Examples

Example 1

Maximize $z = 18x_1 + 16x_2$

subject to

$15x_1 + 25x_2 \leq 375$
$24x_1 + 11x_2 \leq 264$

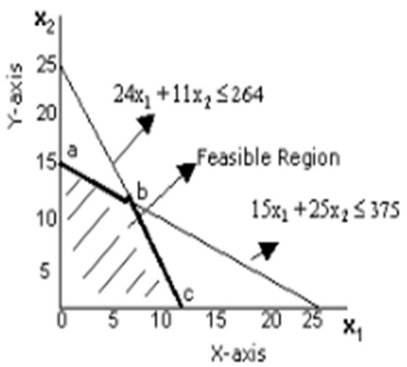
$x_1, x_2 \geq 0$

Solution.

If only x_1 and no x_2 is produced, the maximum value of x_1 is $375/15 = 25$. If only x_2 and no x_1 is produced, the maximum value of x_2 is $375/25 = 15$. A line drawn between these two points (25, 0) & (0, 15), represents the constraint factor $15x_1 + 25x_2 \leq 375$. Any point which lies on or below this line will satisfy this inequality and the solution will be somewhere in the region bounded by it.

Similarly, the line for the second constraint $24x_1 + 11x_2 \leq 264$ can be drawn. The polygon *oabc* represents the region of values for x_1 & x_2 that satisfy all the constraints. This polygon is called the solution set.

The solution to this simple problem is exhibited graphically below.



The end points (corner points) of the shaded area are (0,0), (11,0), (5.7, 11.58) and (0,15). The values of the objective function at these points are 0, 198, 288 (approx.) and 240. Out of these four values, 288 is maximum.

The optimal solution is at the extreme point b, where $x_1 = 5.7$ & $x_2 = 11.58$, and $z = 288$.

Graphical Method in LPP: Example 2

Maximize $z = 6x_1 - 2x_2$

subject to

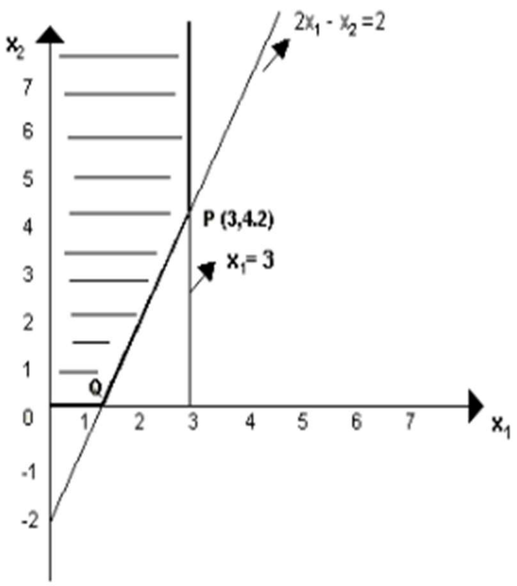
$2x_1 - x_2 \leq 2$
$x_1 \leq 3$

$x_1, x_2 \geq 0$

Solution.

First, we draw the line $2x_1 - x_2 \leq 2$, which passes through the points (1, 0) & (0, -2). Any point which lies on or below this line will satisfy this inequality and the solution will be somewhere in the region bounded by it.

Similarly, the line for the second constraint $x_1 \leq 3$ is drawn. Thus, the optimal solution lies at one of the corner points of the dark shaded portion bounded by these straight lines.



Optimal solution is $x_1 = 3$, $x_2 = 4.2$, and the maximum value of z is 9.6.

SIMPLEX METHOD IN LPP (EXAMPLES)

Find solution using Simplex method
MAX Z = 30x1 + 40x2
subject to
3x1 + 2x2 <= 600
3x1 + 5x2 <= 800
5x1 + 6x2 <= 1100
and x1,x2 >= 0

Solution:
Problem is

Max $Z = 30x_1 + 40x_2$

subject to

$3x_1 + 2x_2 \le 600$

$3x_1 + 5x_2 \le 800$

$5x_1 + 6x_2 \le 1100$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint-1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint-2 is of type ' $\le$ ' we should add slack variable S_2

3. As the constraint-3 is of type ' $\le$ ' we should add slack variable S_3

After introducing slack variables

Max $Z = 30x_1 + 40x_2 + 0S_1 + 0S_2 + 0S_3$

subject to

$3x_1 + 2x_2 + S_1 = 600$

$3x_1 + 5x_2 + S_2 = 800$

$5x_1 + 6x_2 + S_3 = 1100$

and $x_1, x_2, S_1, S_2, S_3 \ge 0$

Iteration-1		C_j	30	40	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_2}$
S_1	0	600	3	2	1	0	0	$\frac{600}{2} = 300$
S_2	0	800	3	(5)	0	1	0	$\frac{800}{5} = 160 \rightarrow$
S_3	0	1100	5	6	0	0	1	$\frac{1100}{6} = 183.3333$
$Z = 0$		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-30	-40 ↑	0	0	0	

Negative minimum $Z_j - C_j$ is -40 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 160 and its row index is 2. So, the leaving basis variable is S_2.

∴ The pivot element is 5.

Entering $= x_2$, Departing $= S_2$, Key Element $= 5$

+ R_2(new) = R_2(old) ÷ 5

+ R_1(new) = R_1(old) - $2R_2$(new)

+ R_3(new) = R_3(old) - $6R_2$(new)

Iteration-2		C_j	30	40	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_1}$
S_1	0	280	1.8	0	1	-0.4	0	$\frac{280}{1.8} = 155.5556$
x_2	40	160	0.6	1	0	0.2	0	$\frac{160}{0.6} = 266.6667$
S_3	0	140	(1.4)	0	0	-1.2	1	$\frac{140}{1.4} = 100 \rightarrow$
$Z = 6400$		Z_j	**24**	**40**	**0**	**8**	**0**	
		$Z_j - C_j$	$-6 \uparrow$	0	0	8	0	

Negative minimum $Z_j - C_j$ is -6 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 100 and its row index is 3. So, the leaving basis variable is S_3.

$\therefore$ The pivot element is 1.4.

Entering $= x_1$, Departing $= S_3$, Key Element $= 1.4$

+ $R_3(\text{new}) = R_3(\text{old}) \div 1.4$

+ $R_1(\text{new}) = R_1(\text{old}) - 1.8 R_3(\text{new})$

+ $R_2(\text{new}) = R_2(\text{old}) - 0.6 R_3(\text{new})$

Iteration-3		C_j	30	40	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	MinRatio
S_1	0	100	0	0	1	1.1429	-1.2857	
x_2	40	100	0	1	0	0.7143	-0.4286	
x_1	30	100	1	0	0	-0.8571	0.7143	
$Z = 7000$		Z_j	**30**	**40**	**0**	**2.8571**	**4.2857**	
		$Z_j - C_j$	0	0	0	2.8571	4.2857	

Since all $Z_j - C_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 100, x_2 = 100$

Max $Z = 7000$

1. Find solution using Simplex method
MAX Z = 3x1 + 5x2 + 4x3
subject to
2x1 + 3x2 <= 8
2x2 + 5x3 <= 10
3x1 + 2x2 + 4x3 <= 15
and x1,x2,x3 >= 0

Solution:
Problem is

Max $Z = 3x_1 + 5x_2 + 4x_3$

subject to

$$2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$

and $x_1, x_2, x_3 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint-1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint-2 is of type ' $\le$ ' we should add slack variable S_2

3. As the constraint-3 is of type ' $\le$ ' we should add slack variable S_3

After introducing slack variables

Max $Z = 3x_1 + 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

subject to

$$2x_1 + 3x_2 + S_1 = 8$$
$$2x_2 + 5x_3 + S_2 = 10$$
$$3x_1 + 2x_2 + 4x_3 + S_3 = 15$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$

Iteration-1		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_2}$
S_1	0	8	2	(3)	0	1	0	0	$\frac{8}{3} = 2.67 \rightarrow$
S_2	0	10	0	2	5	0	1	0	$\frac{10}{2} = 5$
S_3	0	15	3	2	4	0	0	1	$\frac{15}{2} = 7.5$
$Z = 0$		Z_j	0	0	0	0	0	0	
		$C_j - Z_j$	3	5 $\uparrow$	4	0	0	0	

Positive maximum $C_j - Z_j$ is 5 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 2.67 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 3.

Entering $= x_2$, Departing $= S_1$, Key Element $= 3$

R_1(new) = R_1(old) $\div$ 3

R_2(new) = R_2(old)-$2R_1$(new)

R_3(new) = R_3(old)-$2R_1$(new)

Iteration-2		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_3}$
x_2	5	$\frac{8}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$	0	0	---
S_2	0	$\frac{14}{3}$	$-\frac{4}{3}$	0	(5)	$-\frac{2}{3}$	1	0	$\frac{\frac{14}{3}}{5} = 0.93 \rightarrow$
S_3	0	$\frac{29}{3}$	$\frac{5}{3}$	0	4	$-\frac{2}{3}$	0	1	$\frac{\frac{29}{3}}{4} = 2.42$
$Z = \frac{40}{3}$		Z_j	$\frac{10}{3}$	5	0	$\frac{5}{3}$	0	0	
		$C_j - Z_j$	$-\frac{1}{3}$	0	4 ↑	$-\frac{5}{3}$	0	0	

Positive maximum $C_j - Z_j$ is 4 and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 0.93 and its row index is 2. So, the leaving basis variable is S_2.

∴ The pivot element is 5.

Entering $= x_3$, Departing $= S_2$, Key Element $= 5$

$R_2(\text{new}) = R_2(\text{old}) \div 5$

$R_1(\text{new}) = R_1(\text{old})$

$R_3(\text{new}) = R_3(\text{old}) - 4R_2(\text{new})$

Iteration-3		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_1}$
x_2	5	$\frac{8}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$	0	0	$\frac{\frac{8}{3}}{\frac{2}{3}} = 4$
x_3	4	$\frac{14}{15}$	$-\frac{4}{15}$	0	1	$-\frac{2}{15}$	$\frac{1}{5}$	0	---
S_3	0	$\frac{89}{15}$	$\left(\frac{41}{15}\right)$	0	0	$-\frac{2}{15}$	$-\frac{4}{5}$	1	$\frac{\frac{89}{15}}{\frac{41}{15}} = 2.17 \rightarrow$
$Z = \frac{256}{15}$		Z_j	$\frac{34}{15}$	5	4	$\frac{17}{15}$	$\frac{4}{5}$	0	
		$C_j - Z_j$	$\frac{11}{15}$ ↑	0	0	$-\frac{17}{15}$	$-\frac{4}{5}$	0	

Positive maximum $C_j - Z_j$ is $\frac{11}{15}$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 2.17 and its row index is 3. So, the leaving basis variable is S_3.

∴ The pivot element is $\frac{41}{15}$.

Entering $= x_1$, Departing $= S_3$, Key Element $= \frac{41}{15}$

$R_3(\text{new}) = R_3(\text{old}) \times \frac{15}{41}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{2}{3}R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) + \frac{4}{15}R_3(\text{new})$

Iteration-4		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio
x_2	5	$\frac{50}{41}$	0	1	0	$\frac{15}{41}$	$\frac{8}{41}$	$-\frac{10}{41}$	
x_3	4	$\frac{62}{41}$	0	0	1	$-\frac{6}{41}$	$\frac{5}{41}$	$\frac{4}{41}$	
x_1	3	$\frac{89}{41}$	1	0	0	$-\frac{2}{41}$	$-\frac{12}{41}$	$\frac{15}{41}$	
$Z = \frac{765}{41}$		Z_j	3	5	4	$\frac{45}{41}$	$\frac{24}{41}$	$\frac{11}{41}$	
		$C_j - Z_j$	0	0	0	$-\frac{45}{41}$	$-\frac{24}{41}$	$-\frac{11}{41}$	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = \frac{89}{41}, x_2 = \frac{50}{41}, x_3 = \frac{62}{41}$

Max $Z = \frac{765}{41}$